The **topics** and **types of questions** examined in this Achievement Standard. Use this sheet to plan and organise your study so that you cover everything that is required.

3.4 SOLVE EQUATIONS AS 90644

3.4 1. Simultaneous equations

- solving systems of 3 linear equations A

Solve the system of equations:

$$4x+y-2z=-18$$
$$x+y+z=0$$
$$-2x+3z=17$$

- forming and solving a system of 3 linear equations M

A gardener wants to grow three types of plants (A, B, and C) in a garden.

The total cost of purchasing 40 of plant A, 60 of plant B, and 100 of plant C is $3600.

The price of each plant A is three dollars more than the price of each plant C.

The price of each plant B is six dollars more than twice the price of each plant C.

Set up and solve a system of equations to find the cost of each plant C.

- analysing or interpreting the solution or process E
  - may include discussing consistency or non-independence of 3 × 3 systems of linear equations, including geometric representations

Consider the following system of three equations in *x, y* and *z*

$$2x + 4y + 5z = 17$$
$$4x + ay + 3z = b$$
$$8x + 7y + 13z = 40$$

Give values for a and b in the second equation that make this system consistent, but with an infinite set of solutions.

3.4 2. Non-linear equations

- solving a non-linear equation A
  - using the Newton-Raphson method with a given starting value
  - using the bisection method with a given starting value

The equation $x^4-6x+1=0$ has a real root between 0 and 1.

Complete two iterations of either the bisection method, or the Newton-Raphson method, to find an approximation to this root. Show each iterate.

Use $x_0=0.5$ as the starting value if you use the Newton-Raphson method.

Use $x_0=0$ and $x_1=1$ as the starting values if you use the bisection method.

- finding appropriate solutions using either the Newton-Raphson method or the bisection method to improve the approximation to a stated precision for a specified number of iterations M

The equation $0.8x^3-4.2x^2+12.7=0$ has three real roots, all of which lie between – 5 and 5.

Use either the bisection method or the Newton-Raphson method to find the greatest root of the equation $0.8x^3-4.2x^2+12.7=0$.

You must state your starting value(s), show the results of each iteration and give your answer correct to one decimal place.

- giving advantages and disadvantages of the Newton-Raphson method or the bisection method for the problem E
- giving a geometric description of the Newton-Raphson method or the bisection method E

A student wanted to use the Newton-Raphson method to solve the equation $0.8x^3 - 4.2x^2 + 12.7 = 0$

He decided to take $x_o = 0$ as the starting value.

Explain why the use of the Newton-Raphson method, with this starting value, fails to produce a series of iterations that converge to a real root of $0.8x^3 - 4.2x^2 + 12.7 = 0$

3.4 3. Linear programming

- optimising an objective function where the constraints and the objective function are given A

To supplement his income, Marty makes wooden frames for climbing roses, which he then sells to the local garden centres. Marty makes and sells two different-sized frames, a medium one and a large one.

Let x represent the number of medium frames Marty makes and sells in a week, and y represent the number of large frames Marty makes and sells in a week.

The following inequations represent the weekly constraints in the production of Marty's frames:

$$7500x+15\,000y\le 1\,050\,000 \qquad 5x+8y\le 600 \qquad x\ge 20 \qquad y\ge 40$$

(a) Draw these constraints on an axes, and show the feasible region.

Marty's weekly profit ($*P*) from the frames is given by the equation:

$$P=9x+12y$$

(b) Calculate Marty's maximum weekly profit. Justify your answer.

- optimising an objective function, which may require: M
  - forming some constraints
  - forming the objective function
  - rounding the solution in relation to the context
- determining the effect of varying the constraints or objective function of a linear programming problem E
- considering the possibility of multiple solutions to a linear programming problem E